Numerical Simulation of Internal Waves in the Littoral Ocean

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LONG-TERM GOAL

Our long-term goal is to employ numerical simulation to generate accurate predictions of nonhydrostatic internal-tide events, such as large internal waves and solitons, in the littoral ocean.

OBJECTIVES

Our oceanographic-scale objective is to work collaboratively with oceanographers carrying out field-scale experiments to quantify the significant wave events triggered by internal tides, including the nonhydrostatic formation of solitons and their evolution.

Our laboratory-scale simulation objective is to quantify the effects of the breaking instability as well as to study the three dimensional mechanisms of the breaking. These results will inform our field-scale efforts. This objective has been achieved by carrying out laboratory-scale simulations of waves breaking on slopes and in zones where wave characteristics are focused and comparing them to experiments being done by others in our laboratory.

Our numerical objective is to blend a proven field-scale code with large-eddy simulation [LES] and the modeling of domains with irregular boundaries. Our tool is LES in three dimensions and time.

Our numerical analysis objectives include accurate representation of the flow near rough boundaries, creation of improved models for the sub-filter scale [i.e., unresolved] motions, and optimization of the computer code for multiprocessor computer systems.

APPROACH

For simulations of the littoral ocean, we using the UnTRIM field-scale code [Casulli, 1999 a & b and Casulli and Walters, 2000]. It is a finite-volume, nonhydrostatic and free-surface code that employs an unstructured, staggered-grid. This code uses cells composed of Delaunay triangles in the horizontal plane with layers of uniform [but arbitrary] thickness in the vertical. The triangles allow boundary following in plan form, with a variable grid spacing so that one can concentrate grid points over canyons, etc. The thickness of the layers varies from layer to layer. We will install our large-eddy simulation [LES] subfilter-scale [SFS] model to handle turbulence [Katopodes, et. al, 2000 a & b].

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14. ABSTRACT
Our long-term goal is to employ numerical simulation to generate accurate predictions of nonhydrostatic internal-tide events, such as large internal waves and solitons, in the littoral ocean.

15. SUBJECT TERMS

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A major focus of our work is dealing with the nonhydrostatic internal-tide events leading to large nonhydrostatic internal waves and solitons. We will examine the effect of three-dimensional bathymetry on internal tide generation, propagation, and transformation. Our code treats the entire domain and so the evolution from hydrostatic to nonhydrostatic motions is seamless.

Simultaneously, laboratory-scale simulation work has been carried out to examine in detail the physics of breaking internal waves and to test theory against repeatable laboratory experiments being done in the Environmental Fluid Mechanics Laboratory. The primary goal of this work was to quantify the mixing efficiency of laboratory-scale breaking interfacial waves in order to parameterize the effects for our larger scale models. The code used to model these laboratory-scale waves employs many of the features of the Casulli code mentioned above. The code splits the pressure into its hydrostatic and hydrodynamic components. The hydrostatic component consists of the barotropic and the baroclinic components of the pressure, and because of the high speed of the barotropic waves, we model them implicitly with a theta-method. The hydrodynamic pressure is computed with a multigrid method.

WORK COMPLETED

In the past year, we have:

1. created and implemented a conservative, second-order in space and time scalar subroutine for salinity and temperature in the field-scale code.

2. made validation studies of the scalar algorithm and its use in calculating the baroclinic terms in the momentum equations, which are required to generate internal waves.

3. Assembled the grid of Monterey Bay tides and made preliminary tests with a first goal to reproduce the simulations of Rosenfeld, et al. (1999).

4. completed direct numerical simulations of breaking internal waves at the laboratory-scale.

RESULTS

Field-scale simulations: The work with UnTRIM has led us to have the following components in place: code with scalar module and the "realistic" Monterey grid of Rosenfeld, et al. (1999). Our validation studies have included the particularly demanding lock exchange problem [which is driven by the horizontal density differences alone and can only be successfully done with a nonhydrostatic code] and verification that the UnTRIM code produces [in the barotropic mode] the co-oscillating tide behavior of Monterey Bay. Here we show in Figure 1 simulations for the lock exchange problem. These were done with the full three-dimensional code, but at a small scale so that they could be compared to the simulations done in the laboratory-scale part of our project [see below] with a different code. Our UnTRIM code uses the theta method for momentum and either a second-order in time backward scheme or the theta method for scalars; this simulation was run with $\theta = 0.5$ [equivalent to the Crank-Nicolson scheme] in both parts of the code. The domain is 2 m wide by 2 m long by 2 m deep [at its deepest]. The spatial resolution is 0.125 m in the vertical. 528 triangular cells are used and the time step was 0.025 s. The density difference was 30 kg/m^3 with the heavier water [brown] on the left. Here we show the intitial conditions and the result at 3.75 s. This simulation was done with a sloping bottom to test that aspect of the scalar algorithm. The code properly accounts for the actual

2

height of the sides of each bottom cell, yielding precisely the correct horizontal velocities and correct vertical flux in each cell. The bottom line here passes through the sides of the bottom cells.
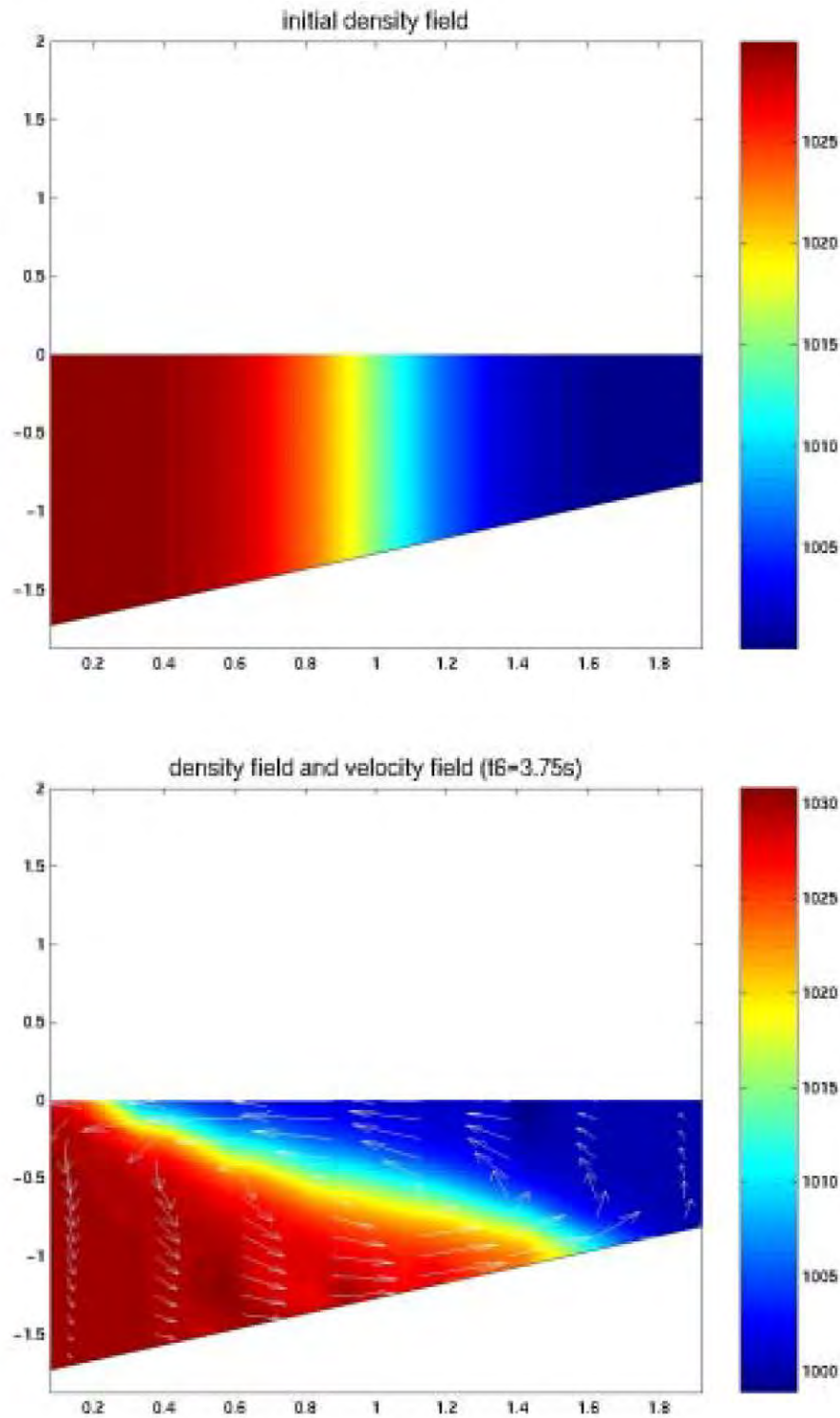

Figure 1. Lock-exchange simulation in 2 m by 2 m horizontal domain and on a sloping bottom [with depths ranging linearly from 2 to 1 m. The scalar concentrations and velocities in the vertical plane are superimposed.

Laboratory-scale simulations: We have developed robust techniques to generate large amplitude interfacial waves in a periodic domain. We used one technique to generate breaking interfacial waves in two and three dimensions in a laboratory scale tank with length L=0.2 m, width W=0.2 m, and depth d=0.3 m, which is the size of the experiments of Troy and Koseff (2000). The interface is located at mid-depth, and we vary the interface thickness to test its effects on the breaking dynamics.

Figure 2 depicts the mean density contour history of a growing two dimensional interfacial wave with an interface thickness of $k\delta = 0.31$, where k is the wavenumber and δ is the interface thickness. At $t/T = 3.4$, where T is the linearized wave period, a shear instability develops, forming Kelvin-Helmholtz billows that lead to wave breakdown. An interfacial wave with this interface thickness reaches a maximum steepness of ka = 0.74 before it breaks, where a is the wave amplitude. Further increasing the interface thickness reduces the strength of the shear instability, thus allowing the wave to reach a greater steepness before it breaks. This steepness asymptotes to the inviscid limit of ka=1.1 (Holyer, 1979), as shown in Figure 3. Figure 3 also shows the effect of background shear, which reduces the maximum steepness before breaking, as is also shown by Thorpe (1978).

The two-dimensional simulations show that progressive interfacial waves always break as a result of a shear instability before a convective instability takes place. Figure 4 depicts the three-dimensional dynamics that ensue after the two dimensional instability takes place. Waves on a thicker interface result in more catastrophic breaking than thinner-interface waves. The thicker interface results in a more energetic Kelvin-Helmholtz instability which energizes the nonlinear cross-stream instability more efficiently, causing almost a complete loss of wave energy after roughly 4.5 wave periods. In Figure 4, a bulk of the wave energy still remains 5 periods after breaking begins. In Figure 5 we show a superposition of the interface density surface, longitudinal vorticity generated in the breaking process, and velocities in a vertical streamwise plane. The blue and white contours depict isosurfaces of constant +/- longitudinalvorticity. The red surface is the mean density surface, and the centerplane depicts the velocity vectors in that plane.
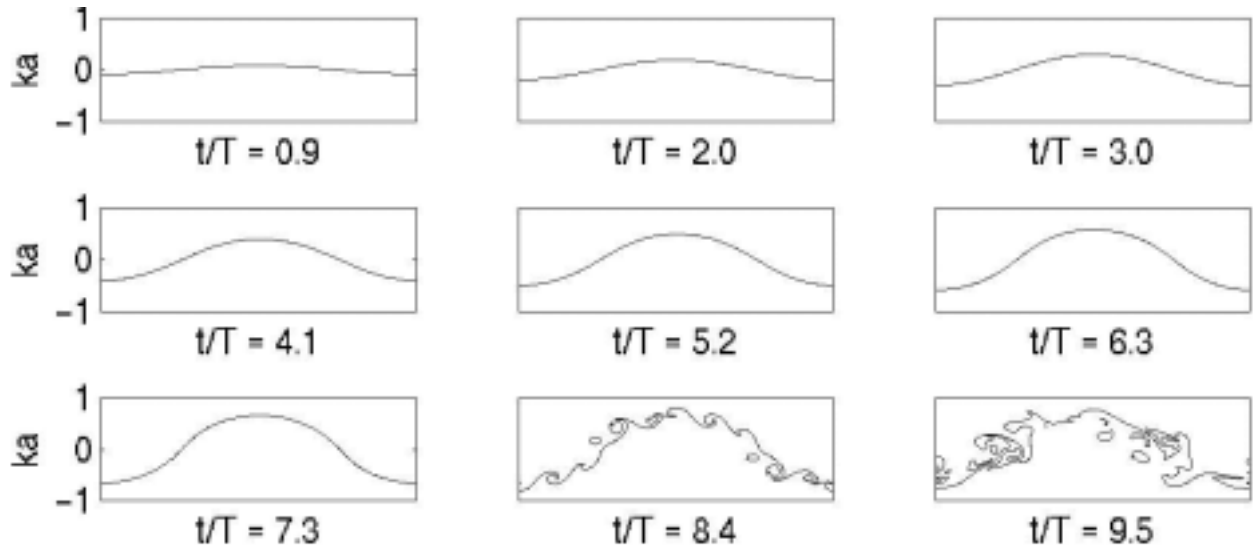

Figure 2. Contour plot of the mean density line at different times during the growth of a forced interfacial wave with $k\delta$ =0.31
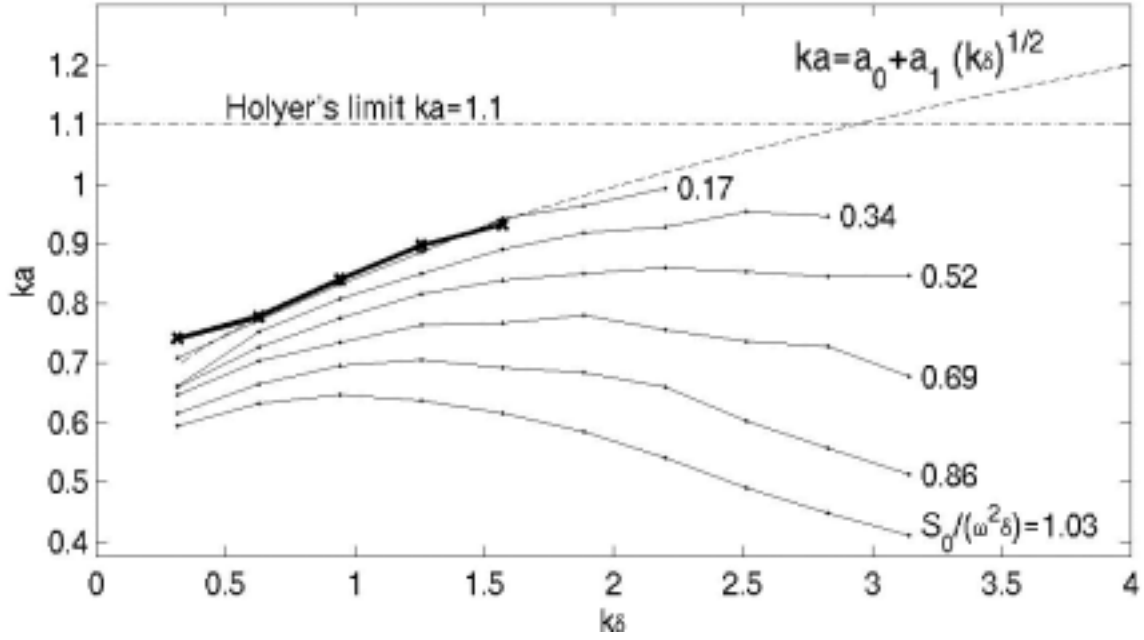
Figure 3. Interfacial wave breaking amplitude as a function of the interface thickness for increasing background shear. The dashed line represents the zero shear scaling approximation, while the solid lines depict results with different background shear, and the thick solid line depicts the zero shear results. $S_0/(\omega^2 \delta)$ is the shearing parameter, a is the wave amplitude, δ is the interface thickness, ω is the wave frequency, and k is the wavenumber.
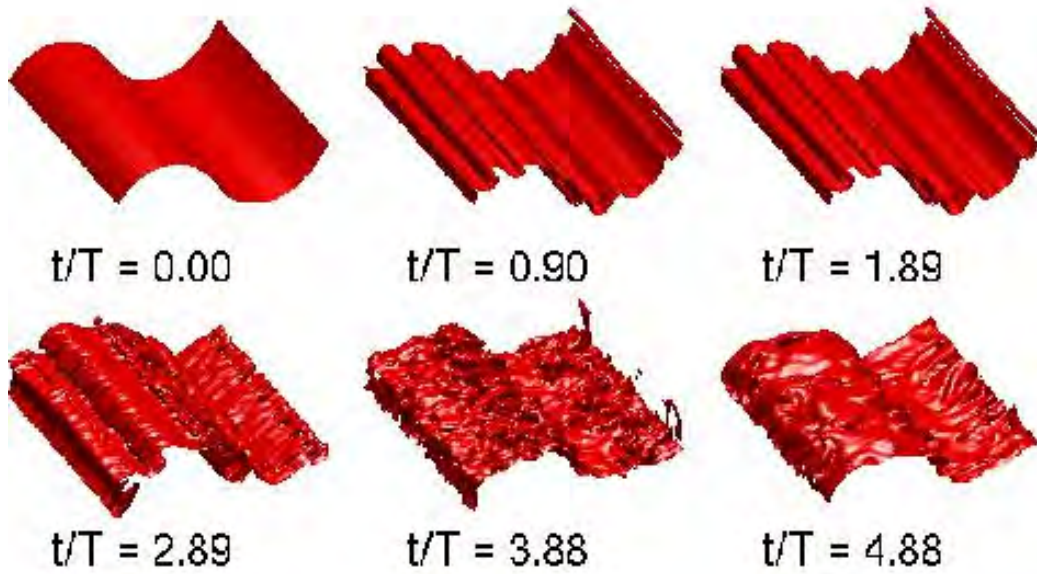

Figure 4. Three dimensional isosurface history of the mean density of a breaking interfacial wave with $k\delta = 0.31$. T corresponds to the linearized wave period.
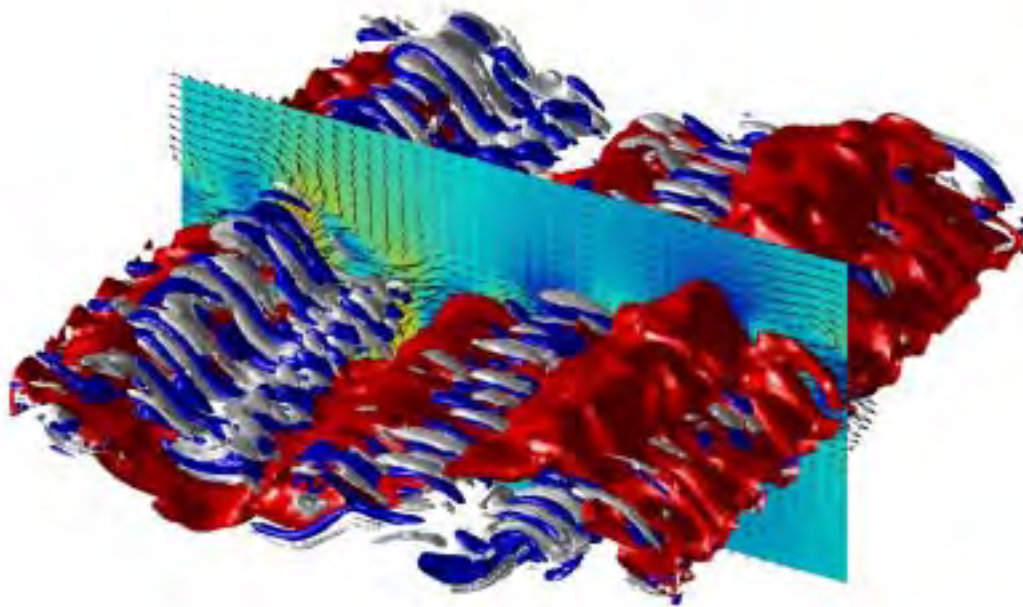
Figure 5. A superposition of the interface density surface, longitudinal vorticity generated in the breaking process,and velocities in a vertical streamwise plane. The blue and white contours depict isosurfaces of constant +/- longitudinalvorticity. The red surface is the mean density surface, and the centerplane depicts the velocity vectors in that plane.

We have shown with this research that the amplitude of a progressive interfacial wave is limited by a shear instability that develops at the interface. Increasing the interface thickness delays this shear instability, but the maximum amplitude is limited to ka = 1.1, which is the inviscid limit and the point at which the interface profile develops a verticality in slope half way between the crest and trough. The initial instability in a breaking interfacial wave is always a two dimensional shear instability, which is followed by a nonlinear cross-stream instability that derives its energy from the Kelvin-Helmholtz billows. Breaking interfacial waves with thicker interfaces dissipate wave energy faster than thinner waves do as a result of the larger and more energetic Kelvin-Helmholtz billows.

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